Spectral Estimation of Clutter for Matched Illumination Waveforms

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Abstract:
The Matched Illumination (MI) technique exploits the difference in spectral characteristic of target and clutter for the design of transmit waveform and receive filter. This results in less interference to the signal processor of a sensor, thereby increasing the Signal to Interference Noise Ratio (SINR). An important aspect of MI is accurate estimation of clutter and target spectra. In this paper we investigate conventional spectral estimation techniques and apply them to the concept of MI. We numerically evaluate the loss in performance of MI due to inaccurate estimation.

Keywords*: Matched Illumination, Spectrum Estimation, Clutter Power Spectral Density, Signal to Interference Noise Ratio.*

I INTRODUCTION

MI technique is based on exploiting different spectral characteristics of returns from target and clutter. The spectral characteristic of the response from target is usually different from that of the clutter. Exploiting this difference will result in selective rejection of clutter thereby increasing Signal to Interference Ratio (SINR) at the output of receive filter (matched filter in conventional radar case). In MI, modulation of transmission pulse and corresponding receive filter are adaptively matched to the environment. The sensor analyzes the environment in real time and based on the estimates of clutter and target spectra generates optimal modulation envelope for transmission and corresponding receive filter. This adaptive feedback loop results in rejection of clutter at receive filter stage and enhances detection of target both in clutter and noise conditions.

The framework of MI has been focus of attention in recent past [1], [2], [3]. It has been reported that MI could potentially provide about 3-5 dB improvement in SINR for a radar, provided that the clutter power spectral density and target spectrum are known accurately at the receiver. A similar study [4] was undertaken in EADS on MI for various target and clutter scenarios. SINR improvements over conventional radars were observed for various target and clutter scenarios. The Ambiguity Function (AF) parameters of MI were also investigated [5], [6], which led to the conclusion that MI has better performance with regard to parameters like accuracy, resolution, side-lobe level, etc.

Another aspect which has been investigated is generating constant amplitude transmit signal, which is typically required in radar transmitters [7], [8], [9]. Many schemes have been investigated that result in real-time computation of constant amplitude waveform even while keeping the desired frequency domain response of transmitted signal, as stipulated by MI technique.

The MI technique requires good estimator of spectra of interference and target. Conventional radar modulations like LFM, NLFM can be used as sounding pulse to determine clutter and target spectra. The estimates of clutter and target spectra are built from the return (a priori knowledge of target position is assumed, as will be the case in track dwells of a multifunction radar). On the basis of the estimated spectra of interference and target, the transmit waveform for the next cycle and corresponding optimal receive filter is determined. Transmit and receive filter can be recursively updated for subsequent cycles of transmission and reception. Accurate estimator of clutter and target are key as inaccurate estimation of clutter and target spectra will result in sub-optimal transmit waveform and receive filter, which will deteriorate the SINR improvement that MI could achieve.

In this paper we address the problem of clutter estimation, incorporate it in MI framework and provide assessment of the loss of performance on account of inaccurate estimator. It is being assumed that target spectrum is known. Estimation of target spectrum will be taken up in future work.

The paper is organized as follows. In section I an overview of MI technique is provided. The section II the candidate spectral estimation techniques are described. Performance of estimation techniques is assessed keeping in view the application for radar scenario. In section III the framework of incorporating spectral estimation techniques to MI is provided. Section IV describes the simulation and results of MI and clutter spectral estimation techniques.

II OVERVIEW OF MI

From the Transmit-Receive radar model in Figure 1, the received signal at the receiver can be expressed as

$$y(t) = h(t) * x(t) + c(t) * x(t) + n(t) \qquad 1)$$

The signal component is

$$z_S(t) = x(t) * h(t) \qquad 2)$$

The interference component is

$$z_I(t) = n(t) + x(t) * c(t) \qquad 3)$$

where $*$ represents convolution, $h(t)$ is the target impulse response. The clutter response $c(t)$ is from a dense background and is spread through-out in time, and manifests at the receiver as self-interference term, $x(t) * c(t)$. The receiver filter response is $r(t)$. The receiver thermal noise is $n(t)$, being a complex-valued, zero-mean additive white Gaussian noise (AWGN) with flat spectrum and power spectral density (PSD) $S_{nn}(f)$, which is non-zero over the entire waveform bandwidth. Let $c(t)$ be a complex-valued, zero-mean Gaussian random process representing an interference component, characterized by the PSD $S_{cc}(f)$.

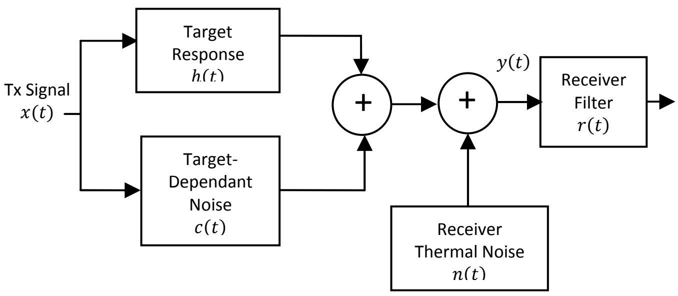


Figure 1 Radar Tx-Rx Model

As described in [10], the SINR optimized transmitted signal $\{X_{SINR}(f)\}$ can be computed by solving

$$X_{SINR}(f) = \max_{X(f)} \int_{f\in\Omega} \frac{|H(f)X(f)|^2}{S_{cc}(f)|X(f)|^2 + S_{nn}(f)} df \qquad 4)$$

The solution is

$$|X_{SINR}(f)|^2 = \max\left(0, \frac{\sqrt{|H(f)|^2 S_{nn}(f)}}{S_{cc}(f)}\left[\mu - \sqrt{\frac{S_{nn}(f)}{|H(f)|^2}}\right]\right) \qquad 5)$$

where μ is the Lagrangian multiplier constant determined from the energy constraint $\int_\Omega |X(f)|^2 df = E$. Note that the self-interference clutter term $\frac{\sqrt{|H(f)|^2 S_{nn}(f)}}{S_{cc}(f)}$ modulates the conventional water-filling solution, $\left[\mu - \sqrt{\frac{S_{nn}(f)}{|H(f)|^2}}\right]$.

II SPECTRAL ESTIMATION

Clutter can be defined as any unwanted radar echo [10]. A vast amount of literature is available on study of different types of clutter encountered in radar. A major focus has been on modelling the probability distribution of various types of clutter. In MI, however, analysis of the spectral characteristics of clutter is of importance.

A simple ground clutter model was analysed in [11] using parametric methods of power spectral estimation. The true PSD or the ground-truth was determined by computing periodogram of 512 samples of a ground clutter model. Various parametric methods of PSD estimation like Maximum Entropy Method (MEM), Autoregressive Moving Average (ARMA), Prony's Energy Spectral Estimation (PESD), least squares method (LSM), Maximum Likelihood method (MLM) were applied on test data lengths of 8 to 64 samples and a comparative analysis was made.

An adaptive filter to reject clutter using an all pole autoregressive model for clutter was derived in [12][12]. The reason stated for choosing an AR model was to reduce the transient (convergence) time to a dwell time (20-30 pulses). The results of the adaptive filter frequency response at the end of 15 adaptive samples were presented.

[11] and [13] stated that since the traditional non-parametric methods like periodogram and Blackman-Tukey involve Fourier transform of a finite number of samples, there is considerable amount of frequency smearing in computed spectrum due to side-lobes of the (sin x)/x function. The frequency resolution is good if the data length is large, but it gets poorer as the data length decreases. A comparative study of all non-parametric and parametric methods was presented in [13] with 64 samples that consisted of three sinusoids of varying magnitude at different frequency locations and a coloured noise process. The non-parametric methods were unable to resolve close lying sinusoids but presence of coloured noise was well indicated while the parametric methods were concluded to be giving accurate estimates of the sinusoids.

In general, the past works emphasize on the use of parametric methods of estimation when the number of samples available is less than about 64 samples. In a typical ground or ship based radar a larger number of clutter dominated samples are expected. In this case non-parametric techniques are expected to give satisfactory results.

Based on non-parametric techniques given in [14], simulations were undertaken to determine the ideal candidate techniques for subject case. Bartlet's and Welch's techniques are particularly suitable as the variance asymptotically reduces with increase in number of samples, whereas in other techniques like periodogram the variance remains unchanged.

III SPECTRAL ESTIMATION IN MI

The next task is to incorporate the preferred method for clutter spectral estimation into the MI framework as developed in [4] and analyse the performance degradation in SINR because of the use of the estimate of clutter spectrum in place of the actual or true spectrum.

To incorporate the clutter power spectrum estimation into the MI framework, the working of the system has been divided into two parts:

- **Learning cycle:** Linear frequency modulation (LFM) signal is used for sounding the channel. Clutter PSD is estimated from single or multiple transmissions of LFM. LFM is quite common in modern radars and taken as a reasonable choice for probing the channel. In future other modulation schemes like NLFM, etc will also be investigated for probing the channel.
- **MI cycle:** Using the estimated clutter PSD, MI signal and the corresponding receiver filter are generated. MI signal as generated is transmitted and SINR is computed after the receiver filter

Learning cycle

Figure 1 shows a block diagram of transmit-receive chain. In the first transmission-reception cycle, let $x(t)$ be linear frequency modulation (LFM) signal with finite-energy and Fourier transform $X(f)$.

Also as the samples for clutter estimation are chosen without the influence of target, type of target will have no effect in estimating clutter.

The power spectrum of interference is

$$L(f) = S_{nn}(f) + |X(f)|^2 S_{cc}(f) \qquad 6)$$

The task is to estimate $\hat{L}_{lfm}(f)$, using the Welch's method of spectrum estimation. Assuming that $S_{nn}(f)$ is known and using $\hat{L}_{lfm}(f)$, the estimate of $S_{cc}(f)$ is computed as

$$\hat{S}'_{cc}(f) = \frac{\left|\hat{L}_{lfm}(f) - S_{nn}(f)\right|}{|X(f)|^2} \qquad 7)$$

$\hat{L}_{lfm}(f)$ is impacted by the variance of the estimator that may result in inaccurate estimate of $S_{cc}(f)$. The transmitted energy is

$$E_x = \int |X(f)|^2 df \qquad 8)$$

To estimate $\hat{S}'_{cc}(f)$, the clutter return component ($|X(f)|^2 S_{cc}(f)$) has to be higher than the noise component $S_{nn}(f)$. Hence $\hat{L}_{lfm}(f)$ and $\hat{S}'_{cc}(f)$ are computed at higher transmitted energy. The estimate could be averaged over one or multiple M number of transmissions

$$\hat{S}_{cc}(f) = \frac{1}{M}\sum_{i=1}^{M} \hat{S}'^{(i)}_{cc}(f) \qquad 9)$$

In the current framework the $\hat{S}_{cc}(f)$ is considered valid for subsequent MI cycles, though it is possible to recursively update $\hat{S}_{cc}(f)$.

MI cycle

The MI transmit signal is computed by using Eq. (5) replacing $S_{cc}(f)$ with the estimated value $\hat{S}_{cc}(f)$, keeping energy in $X(f)$ and $\hat{X}_{MI}(f)$ same. This signal is transmitted in the next step, i.e. MI cycle

$$\left|\hat{X}_{MI}(f)\right|^2 = \max\left[0, \hat{B}(f)(\mu' - D(f))\right] \qquad 10)$$

where,

$$\hat{B}(f) = \frac{\sqrt{|H(f)|^2 S_{nn}(f)}}{\hat{S}_{cc}(f)} \qquad 11)$$

$$D(f) = \sqrt{\frac{S_{nn}(f)}{|H(f)|^2}} \qquad 12)$$

And to keep the energy of the LFM signal and the MI signal same, i.e.

$$\int |X(f)|^2 df = \int \left|\hat{X}_{MI}(f)\right|^2 df \qquad 13)$$

μ' needs to be computed by 1-D search methods [1]. In our experiment, the well-known root-finding algorithm called secant method [15] has been used to compute μ' using Eq. (13) as the cost function.

Computing $\hat{L}(f)$ for the next iteration (MI cycle),

$$\hat{L}(f) = \left|\hat{X}_{MI}(f)\right|^2 \hat{S}_{cc}(f) + S_{nn}(f) \qquad 14)$$

Redesigning receiver filter for next iteration (MI cycle),

$$\hat{R}(f) = k\left[\frac{\hat{X}_{MI}(f) H(f) e^{-j2\pi f to}}{\hat{L}(f)}\right]^* \qquad 15)$$

In the second transmission-reception cycle, the transmitted signal $\hat{x}(t)$ is the MI waveform computed above and $\hat{r}(t)$ is the corresponding receiver filter. This signal is subjected to the same environment as described in the previous cycle. The output at the receiver is

$$\hat{y}(t) = \hat{r}(t) * [n(t) + \hat{x}_{MI}(t) * c(t) + \hat{x}_{MI}(t) * h(t)] \qquad 16)$$

Signal-to-interference ratio is given by

$$(SINR)_{to} = \frac{\left|\int_{-\infty}^{\infty} \hat{X}_{MI}(f)\hat{R}(f)H(f)e^{-j2\pi f to} df\right|^2}{\int_{-\infty}^{\infty} \left|\hat{R}(f)\right|^2 \left\{\left|\hat{X}_{MI}(f)\right|^2 S_{cc}(f) + S_{nn}(f)\right\} df} \qquad 17)$$

Putting value of $\hat{R}(f)$ from Eq. 15) and solving, we get

$$(SINR)_{to} = \qquad 18)$$

$$\frac{\left|\int_{-\infty}^{\infty} \frac{\left|\hat{X}_{MI}(f)\right|^2 |H(f)|^2}{\left[\left|\hat{X}_{MI}(f)\right|^2 \hat{S}_{cc}(f) + S_{nn}(f)\right]^*} df\right|^2}{\int_{-\infty}^{\infty} \frac{\left|\hat{X}_{MI}(f)\right|^2 |H(f)|^2}{\left|\left[\left|\hat{X}_{MI}(f)\right|^2 \hat{S}_{cc}(f) + S_{nn}(f)\right]^*\right|^2} \left\{\left|\hat{X}_{MI}(f)\right|^2 S_{cc}(f) + S_{nn}(f)\right\} df}$$

IV SIMULATION, RESULTS & CONCLUSION

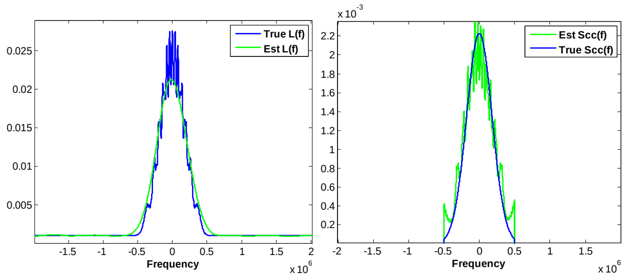
The clutter PSD $S_{cc}(f)$ is assumed to be Gaussian and present in all range cells (without considerations of variation in return strength due to range). The noise is assumed to be additive white Gaussian noise (AWGN). CNR (clutter to noise ratio) and TNR (target to noise ratio) are both assumed to be 0 dB.

The simulation is undertaken to estimate drop in SINR with respect to a case where true PSD of clutter is assumed to be known. The comparison is done by estimating clutter by

- Varying the number of samples for estimation
- Varying the transmitted energy
- Varying the number of learning cycle iterations

In the simulation, about 9000 samples are available for interference estimation. Welch's method with 50 samples segment length, Hamming window and 50% overlap is used to estimate $L(f)$. In the current simulation the $X(f)$ generated is not subjected to constant amplitude constraint.

Figure are the plots for desired and estimated $L(f)$ and $S_{cc}(f)$ for LFM transmit signal. $\hat{S}_{cc}(f)$ is calculated by averaging $\hat{S}'_{cc}(f)$ over two transmissions or iterations (M=2) as per Eq. (9). From the illustration, it is observed that in spite of presence of perturbations due to estimation errors, the estimates are close approximation of the 'true' values of $L(f)$ and $S_{cc}(f)$.



(a) (b)

Figure 2. (a) True $L(f)$ (blue) and $\hat{L}_{lfm}(f)$ (green) (b) True $S_{cc}(f)$ (blue) and $\hat{S}_{cc}(f)$ (green) for 9000 samples and 2 learning cycle iterations

The simulations were undertaken for two different types of targets. In the first case, a point target is assumed, with a flat spectrum in the bandwidth of transmission. In the second case an extended stochastic target was modeled with Gaussian spectrum within the bandwidth of transmission which is fairly separated from the clutter spectrum. Figure 3 shows extended target, clutter and noise PSDs in the bandwidth of 1 MHz.

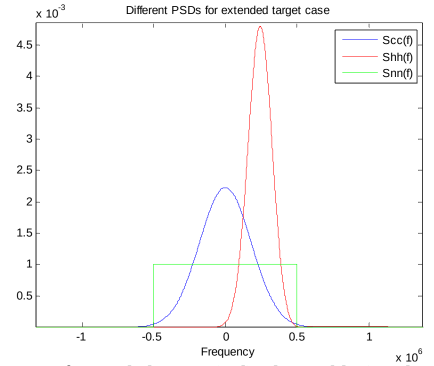


Figure 3. PSDs of extended target (red), clutter (blue) and noise (green)

Table 1 Comparison of SINR values for different Tx-Rx models (TxEnergy=4000) (from Fig. 4)

Type of Target → / Tx-Rx Model ↓		Point Target	Extended Target
MI + OptFilter		62.57	85.85
MI + OptFilter (Estimate)	100 samples	60.78	85.51
	3000 samples	61.71	85.74
Chirp + Matched Filter		58.36	79.84

Table 2 SINR vs number of samples for estimation – Point Target (from Fig. 5)

No of samples	100	500	1000	3000	5000
Desired SINR	58.85				
1 iteration	38.8	49.7	58.28	58.38	58.4
2 iterations	57.4	58	58.47	58.38	58.4
5 iterations	58.1	58.3	58.4	58.4	58.46

Error! Reference source not found.shows variation of SINR with TxEnergy. TxEnergy is given by equation 8) in dB with respect to the noise floor assumed to be 0 dB. **Error! Reference source not found.** (a) shows the SINR plots where $\hat{S}_{cc}(f)$ is computed using 100 samples while in **Error! Reference source not found.**4 (b) it is computed using 3000 samples. In each of these figures, multiple SINR plots represent $\hat{S}_{cc}(f)$ being computed with one or more transmission iterations as per equation (9). It is evident from the figures that as the clutter PSD estimation improves with increase in the number of samples, performance in terms of SINR also improves with increase in the number of samples. Also there is further improvement in SINR if the number of transmission iterations for estimation increases.

tabulates the values from **Error! Reference source not found.**4 (a) and (b) for point and extended targets respectively at a fixed transmit energy.

Figure shows SINR variation with number of samples for clutter estimation when the target is point. The set of sample lengths for which SINR has been computed are 100, 500, 1000, 2000, 3000, 4000 and 5000. The TxEnergy has been kept fixed at 1200 for computing the SINR values. The desired SINR is 58.85 dB. Different plots of SINR have been computed using 1, 2 and 5 transmission iterations for clutter estimation. **Error! Reference source not found.** tabulates the values from Figure5 for reference.

Table 3 Summary table - SINR drop at fixed TxEnergy for Point Target (from Fig. 4)

summarizes the performance in terms of SINR drop with small and large number of samples for estimation with one and two transmission iterations at a fixed TxEnergy.

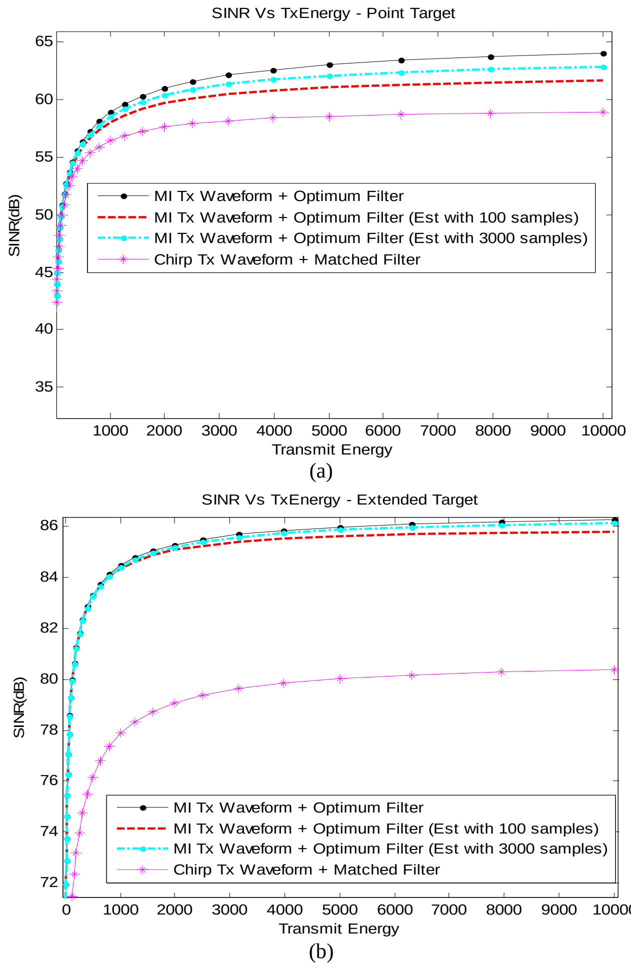


Figure 4. SINR Vs. TxEnergy with (a)100 samples and 3000 samples for estimation. Color coding of the plots is: MI with optimal Rx (black), MI with optimal Rx with estimation (red), Chirp with Matched filter (conventional) (magenta)

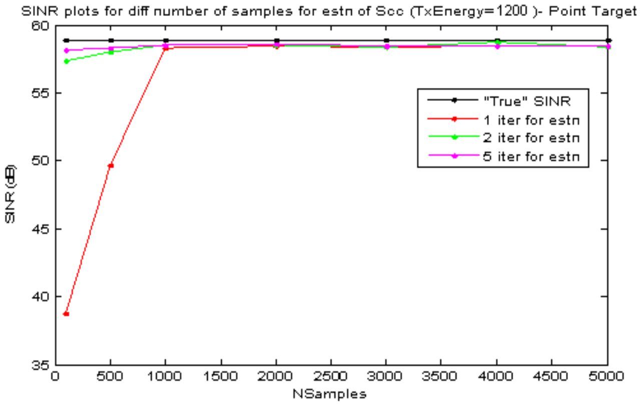


Figure 5. SINR plots for different number of samples for estimation (TxEnergy=1200). Color coding of the plots: Desired (black), 1 iteration (red), 2 iterations (green), 5 iterations (magenta)

Table 3 Summary table - SINR drop at fixed TxEnergy for Point Target (from Fig. 4)

No of samples	**100**	**3000**
1 iteration	20 .05	0.47
2 iterations	1.45	0.47

Based on the above results, we conclude that the techniques such as Welch provide reasonable performance in estimation of clutter PSD. The eventual loss of SINR with real-time clutter estimation is not very significant, provided sufficient samples are available to estimate clutter or estimation can be averaged over multiple transmissions.

REFERENCES

[1] Romero, R.A., Bae, J. and Goodman, N.A., "Theory and application of SNR and mutual information matched illumination waveforms", Aerospace and Electronic Systems, IEEE Transactions on, vol. 47, no. 2, pp. 912--927, 2011.

[2] Bae, J. and Goodman, N.A., "Evaluation of modulus-constrained matched illumination waveforms for target identification", Radar Conference, 2010 IEEE, pp. 871--876, 2010.

[3] Pillai, S.U. and Oh, H.S. and Youla, D.C. and Guerci, J.R., "Optimal transmit-receiver design in the presence of signal-dependent interference and channel noise", Information Theory, IEEE Transactions on, vol. 46, no. 2, pp. 577 - 584, 2000.

[4] Shirodkar, S., Santra, A., Jadia, K., and Srinivasan, R., “Report on Joint Tx Waveform - Rx Filter optimization for Cognitive Radar”, EADS Doc. No. - CAIE-03-COGN-SYS-EDO-AR-02, Nov, 2012.

[5] Santra, A., Srinivasan, R., Jadia, K., and Guillaume, A., “Ambiguity Functions of Matched Illumination Radar Signals”, submitted to IRSI’13 for review.

[6] Santra, A., Srinivasan, R., and Jadia, K., “Report on Ambiguity Function Analysis for MI Waveforms”, EADS Doc. No. - CTO/IW/IOSA-2013/004, March, 2013.

[7] Pillai, S.U., Li, K.Y. and Beyer, H., "Reconstruction of constant envelope signals with given Fourier transform magnitude", Radar Conference, 2009 IEEE, pp. 1--4, 2009.

[8] Kay, S., "Optimal signal design for detection of Gaussian point targets in stationary Gaussian clutter/reverberation", Selected Topics in Signal Processing, IEEE Journal of, vol. 1, no. 1, pp. 31--41, 2007.

[9] Bell, Mark R., "Information theory and radar waveform design", Information Theory, IEEE Transactions on, vol. 39, no.5, pp. 1578--1597, 1993.

[10] Skolnik, M.I, “Introduction to Radar Systems”,New York: McGraw Hill, 1980.

[11] Prabhu K.M.M., Bhoopathy Bagan K.,”Non-Linear spectral-estimation methods for ground clutter analysis”, Electronics & Communication Engineering Journal, IEE, July/August, 1989.

[12] Bowyer, D.E., Rajasekaran, P. K., Gebhart, W. W., “Adaptive Clutter Filtering Using Autoregressive Spectral Estimation“, IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-15, No. 4, July, 1979.

[13] Kay, S. M., Marple, S. L, “Spectrum Analysis – A Modern Perspective”, Proceedings of the IEEE, Vol. 69, No. 11, November, 1981.

[14] Hayes, M. H., “Statistical Digital Signal Processing and Modelling”, New Delhi: John Wiley & Sons, 2002.

[15] Weisstein, Eric W., "Secant Method", from *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/SecantMethod.html

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